

22. YouBessy

3.2.2021

---

$$\begin{aligned} e^z &= \sum_{n \geq 0} \left( \frac{z^n}{n!} \right)^r \\ &= \sum_{n \geq 0} \frac{z^n}{n!} z^{n-1} \\ &= \sum_{n \geq 1} \frac{z^{n-1}}{(n-1)!} \cdot \sum_{n \geq 0} \frac{z^n}{n!} \\ &= e^z. \end{aligned}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i},$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}.$$

Multiplication:

$$e^{z+w} = e^z \cdot e^w.$$

Dann:

$$\begin{aligned} \exp(it) &= \sum_{n \geq 0} \frac{(it)^n}{n!} \\ &= \sum_{n \geq 0} (-1)^n \frac{t^{2n}}{(2n)!} + i \sum_{n \geq 0} (-1)^n \frac{t^{2n+1}}{(2n+1)!} \\ &= \cos(t) + i \cdot \sin(t). \end{aligned}$$

Für alle  $t$ :

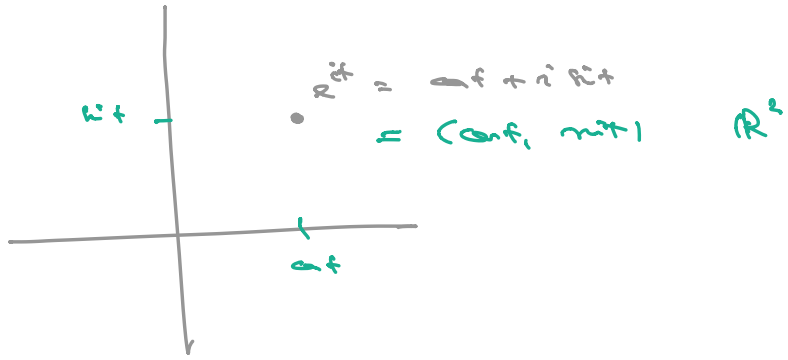
$$e^{it} := \exp(it) = \cos t + i \sin t$$

$t = \pi$ :

$$\begin{aligned} e^{i\pi} &= -1 \\ \Leftrightarrow e^{i\pi} + 1 &= 0. \end{aligned}$$

$$\mathbb{R}^{2i} \ni \mathbb{C} \sim (\mathbb{C}^i, \mathbb{R}^i) \hookrightarrow \mathbb{R}^n$$

A



$$r_{it}^{ci} = p_t + i \cdot r_{it}$$

$$r_{i(S_{t+1})}^{ci} = \underbrace{p(S_{t+1})}_t + i \cdot \underbrace{r_{i(S_{t+1})}}_t$$

$$\begin{aligned} r_{i(S_{t+1})}^{ci} &= (p_t + i \cdot r_{it}) (p_t + i \cdot r_{it}) \\ &= \underbrace{p_t \cdot p_t - r_{it} \cdot r_{it}}_t + i \cdot \underbrace{(r_{it} \cdot p_t + p_t \cdot r_{it})}_t \end{aligned}$$

Also:

$$p(S_{t+1}) = p_t \cdot p_t - r_{it} \cdot r_{it} + \dots$$

$$\begin{aligned} \underbrace{(p_t + i \cdot r_{it})}_t &= \underbrace{(r_{it}^{ci})}_t \\ &= p_{it}^{ci} \\ &= \underbrace{p_t}_t + i \cdot \underbrace{r_{it}}_t \end{aligned}$$

$$\begin{cases} p_{it}^{ci} = p_t + i \cdot r_{it} \\ p_{it}^{ci} = p_t - i \cdot r_{it} \end{cases}$$

$$p_t = \frac{p_{it}^{ci} + p_{it}^{ci}}{2} = p_{it}^{ci}$$

$$r_{it} = \frac{p_{it}^{ci} - p_{it}^{ci}}{2i} = r_{it}^{ci}$$

$$z \neq 0 : \quad z = (x \cdot y) ,$$

$$\vartheta = \frac{y}{x} \in \mathcal{S} \quad : \quad (0 \neq 1)$$

$$\text{Dann} \quad \vartheta = r^{i\varphi} , \quad \varphi \in [0, 2\pi)$$

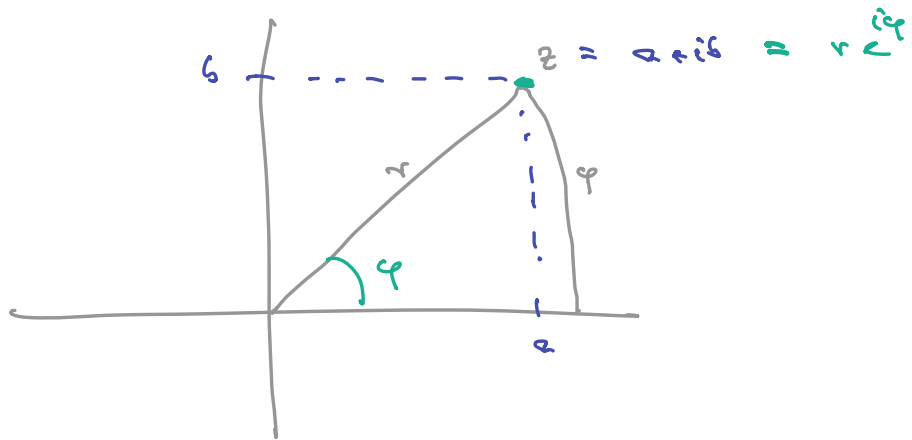
$$\text{Also:} \quad z = (x \cdot \vartheta) = (x \cdot r^{i\varphi})$$

$$= r x^{i\varphi} , \quad r = |x| > 0.$$

$$\text{Complexwert:} \quad |z| = (r \cdot \underbrace{|x^{i\varphi}|}_1) = r ,$$

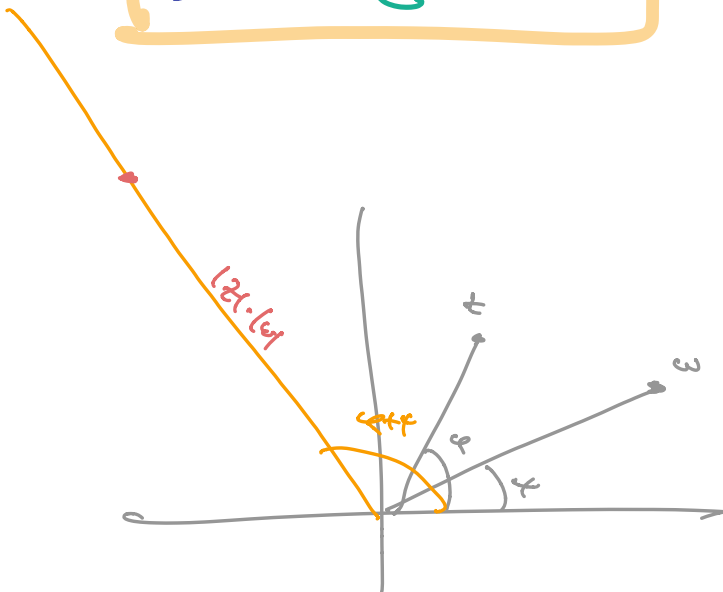
$$z = r x^{i\varphi} = r x^{i(\varphi + 2\pi n)} , \quad n \in \mathbb{Z}$$

$$= r x^{i\varphi} \cdot \underbrace{x^{2\pi n i}}_1 .$$



Ans:  $z = r e^{i\phi}$ ,  $e = a + ib$

$$z = r e^{i(\phi + \pi)}$$



Given:  $z = 1$  find  $z^n$ .

Let:  $z = r e^{i\phi}$

Given:  $z^n = r^n e^{i n \phi} = 1$

Given:  $r = 1$  ✓

So:  $r e^{i n \phi} = 1$

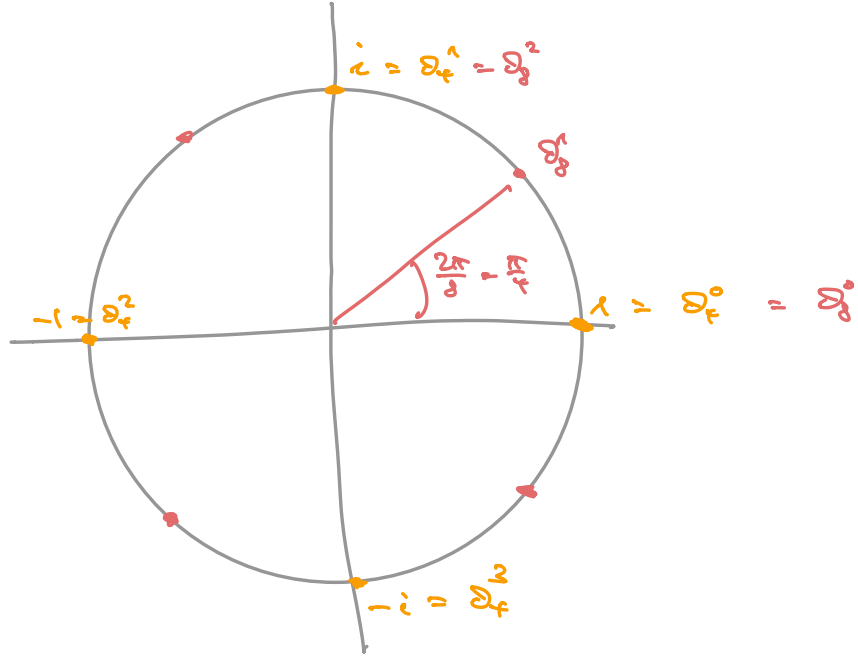
$$n\phi = 2\pi k, \quad k \in \mathbb{Z}$$

$$\Leftrightarrow \phi = 2\pi \frac{k}{n}$$

$$z = r e^{2\pi i \frac{k}{n}}, \quad k = 0, 1, \dots, n-1$$

$$= e^{2\pi i \frac{k}{n}}$$

$$= \left( e^{2\pi i \frac{1}{n}} \right)^k = \omega_n^k$$



Dem:

$$v_1 = r \begin{pmatrix} 1 \\ 1 \end{pmatrix} \neq 0$$

$$v_2 = r \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$v_3 = r \begin{pmatrix} 1 \\ i \end{pmatrix} = r \begin{pmatrix} 1 \\ 0 \\ 0 \\ i \end{pmatrix}$$

Dem

Die Determinant:

$$\begin{pmatrix} \lambda - 1 & 0 \\ 0 & \lambda - 1 \end{pmatrix} = \begin{pmatrix} \lambda - 1 & 0 \\ 0 & \lambda - 1 \end{pmatrix} = 1$$

$$\lambda = 1 \quad \text{I} \quad v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



$$f' = f, \quad f(0) = 1$$

$$f'' = -f,$$

$$f(0) = 0, \quad f'(0) = 1$$

...

$$f'' = f,$$

— " —

$e^{xt}$ ,  $e^{-xt}$  sind Lösungen, so sind  
 $e^{xt} + e^{-xt}$ .

D:

$$\begin{cases} x + 0 = 0 \\ x - 0 = 1 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} \\ x = -\frac{1}{2} \end{cases} \therefore \frac{e^{xt} - e^{-xt}}{2}$$

$$\text{SüR } f = \sum_{k=0}^{\infty} \frac{t^{2k}}{(2k)!} = \frac{e^{xt} + e^{-xt}}{2}$$

$$\text{NüR } f = \sum_{k=0}^{\infty} \frac{t^{2k}}{(2k)!} = \frac{e^{xt} + e^{-xt}}{2}$$

$$\cos^2 t - \sin^2 t$$

$$= \frac{e^{2t} + 2 + e^{-2t}}{4} - \frac{e^{2t} - 2 + e^{-2t}}{4}$$

$$= \frac{2}{4} + \frac{2}{4}$$

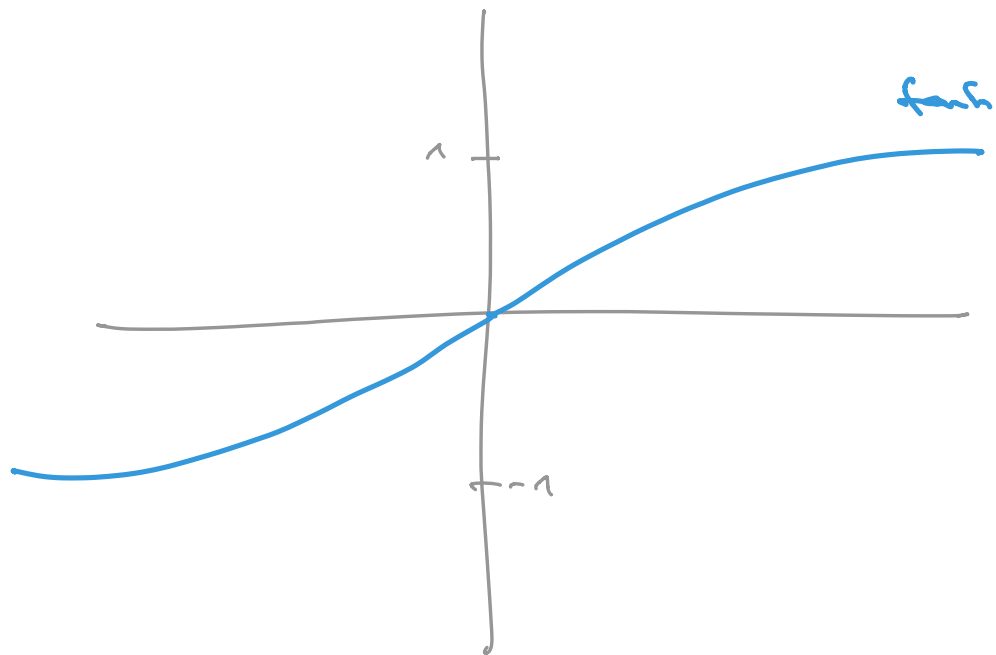
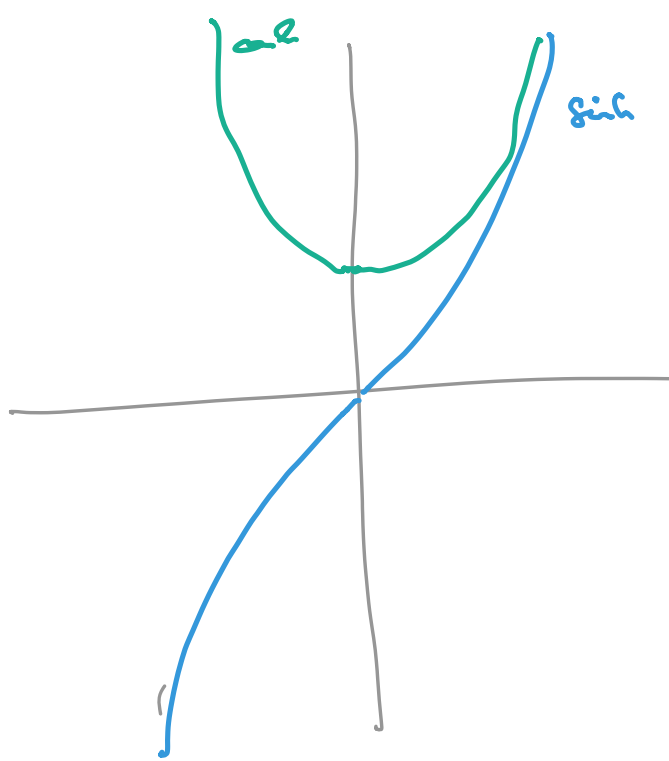
$$= 1$$

$$\sin(it) = \frac{e^{it} - e^{-it}}{2i} \quad (z = it)$$

$$= \frac{1}{2i} (e^{-t} - e^t)$$

$$= \frac{1}{i} \frac{e^t - e^{-t}}{2}$$

$$= -i \sin t$$



Bemerkung:

$$t = \frac{r_1 + r_2}{2} \quad \text{w.} \quad t$$

$$\text{(I)} \quad 2t = r_1 + r_2$$

$$\text{(II)} \quad r_1^2 + 1 = 2t r_1$$

$$\text{(III)} \quad \boxed{r_1^2 - 2t r_1 + 1 = 0} \quad (r_1)$$

$$r_1 = t \pm \sqrt{t^2 - 1} \quad \geq 1$$

für  $t$

$$r_1 = t + \sqrt{t^2 - 1}$$

$$r_2 = \frac{1}{t + \sqrt{t^2 - 1}}$$

